Mathematics Curriculum – Middle Level

May 2007

Introduction

Mathematics Curriculum

At Baker Demonstration School, we believe that all students are capable of learning and applying mathematics. We develop understanding using methods that allow children to continue to construct their knowledge in a social context. Our classroom groups are heterogeneous, and our students do much of their classroom work in clusters of three to five students. We emphasize the importance of verbal and written communication about mathematical ideas and of recognizing that there is always more than one way to solve a math problem. We draw from an assortment of text resources to meet the diverse needs of our students. We encourage students to learn from each other and to develop confidence in their own unique way of looking at the world. All of our students keep journals and reflections about the development of their mathematical ideas.

Teachers facilitate learning for understanding by:

- **Crafting, adapting and enriching solid instructional plans** which align with NCTM standards and meet individual students' needs.
- **Motivating students** to collaborate with and help one another.
- **Guiding individual students** who need expert assistance to be challenged to think more deeply or to consider a higher degree of complexity in their problem solving process, as well as students who need greater support to build understanding.
- **Presenting content and strategies** using a variety of methods acknowledging diverse learning styles and encouraging terminology, definitions, notation, concepts and skills to emerge in the learning process.
- **Designing assessment** that promotes student learning and assists teachers in making instructional decisions.
- **Providing a variety of mathematical tools** such as measuring devices, calculators, manipulatives, games, charts, reference books, etc.
- **Setting high expectations** for all students appropriate to their development, learning and experience.
- **Creating strong links between mathematical ideas** so that students' understanding and knowledge deepens and their ability to apply mathematics expands.
- **Assigning authentic mathematical tasks** to introduce important mathematical ideas and to engage and challenge students intellectually.

Students develop problem solving and critical thinking capabilities by:

- **Constructing** solutions using authentic problems that go beyond computation.
- **Communicating** using mathematical language to describe the problem solving process and to develop metacognitive thinking.
- **Investigating** open-ended, complex mathematical problems with confidence.
- **Reflecting**, refining and exploring conjecture on the basis of evidence and using a variety of reasoning and proof to confirm or disprove those conjectures.

- **Improving** from evaluation that represents a comprehensive picture of their understanding through performance on projects, tests, class work and homework, as well as from reflecting on their own processes and understanding.

- **Developing** confidence and perseverance when tackling difficult problems, demonstrating flexibility in exploring mathematical ideas, and embracing alternative solution paths.

6th Grade Number and Operations:

Concepts	**Skills and Processes**
Fractional Reasoning	• Explore area models to develop understanding of fractional operations • Use benchmark fractions, decimals and percents to refine estimation skills
Place Value	• Review decimal place value • Develop strategies for identifying and counting decimal place value
Number Operations	• Refine and review computational skills with whole numbers • Develop estimation skills with whole number operations • Develop computational skills with percents • Apply Symbolic representation of fraction operations • Find equivalent fractions • Reduce fractions to lowest terms • Develop and analyze algorithms for computing with fractions, decimals and percents and develop fluency in their use • Use pie charts and circle graphs to find and use percents of totals greater than or less than 100 • Refine estimation skills with decimals by using rounding techniques to estimate sums and differences of decimal numbers • Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods
Number Theories	• Refine understanding of prime and composite numbers • Make factor trees to find prime factorization of numbers • Use factors, multiples, prime factorization and relatively prime numbers to solve problems • Explore Pascal's triangle to find number patterns
Quantitative Analysis	• Order fractions and decimals • Work flexibly with fractions, decimals and percents to solve problems
Money	• Develop strategies for estimating discount prices • Develop strategies for estimating tax and tips

6th Grade Geometry and Measurement:

Concepts	Skills and Processes
Two-Dimensional Geometry	• Measure polygon sides and angles • Is familiar with assorted two-dimensional objects, including squares, triangles, other polygons and circles • Develop understanding of characteristics of polygons and the relationships between different kinds of polygons • Apply polygon properties to solve problems • Use mathematical properties of polygons to investigate situations in society and nature • Infer whether or not a shape can tile a surface
Area and Perimeter	• Measure area and perimeter of figures with straight edges • Estimate perimeter and area of figures with curved or irregular shapes • Organize and display data in a systematic way • Develop area formulas for rectangles, triangles, parallelograms and circles • Apply these formulas to different problems • Explore the notion of pi as a ratio • Use a variety of linear and square units appropriate for different situations
Transformational Geometry	• Investigate line and rotational symmetry
Spatial Visualization	• Represent three-dimensional figures in two dimensions • Investigate properties and limitations of two-dimensional representations of three-dimensional objects • Model and represent three-dimensional objects • Use cube models and isometric dot paper to create representations of three-dimensional objects from different perspectives • Develop and refine ability to judge which two-dimensional representation of a three-dimensional object will be most useful in solving a particular problem • Develop an understanding of the limits of a representation

6th Grade Probability and Statistics:

Concepts	Skills and Processes
Probability	• Conduct trials to determine experimental probability • Organize and record outcomes • Analyze games or situations to determine theoretical probability • Interpret experimental and theoretical probabilities to determine whether outcomes are equally likely or biased • Use models to help determine probabilities • Use probabilities to develop strategies for playing games or acting in situations to maximize chances of a favorable outcome occurring

6th Grade Communication of Mathematical Ideas:

Concepts	Skills and Processes
Mathematical Language	• Use mathematical language and representations with appropriate accuracy, including numerical tables and equations, simple algebraic equations and formulas, charts, graphs, and diagrams • Use mathematical language to make complex situations easier to understand
Organization	• Organize work, explain facets of a solution orally and in writing • Label drawings • Use other techniques to make meaning clear to the audience
Reasoning	• Exhibit developing reasoning abilities by justifying statements and defending work
Understanding	• Show understanding of concepts by explaining ideas not only to teachers and assessors but to fellow students or younger children
Reading Comprehension	• Comprehend mathematics from reading assignments and from other sources

6th Grade Problem Solving and Abstract Reasoning:

Concepts	Skills and Processes
Formulation	• Formulate and solve a variety of meaningful problems • Extract pertinent information from situations • Determine what additional information is needed

Concepts	**Skills and Processes**
Implementation	• Use and invent a variety of approaches and understand and evaluate those of others • Invoke problem solving strategies, such as illustrating with sense-making sketches to clarify situations or organizing information in a table • Determine how to break a problem into simpler parts • Solve for unknown or undecided quantities using algebra, graphing, sound reasoning and other strategies • Integrate concepts and techniques from different areas of mathematics • Work effectively in teams when the nature of the task or the allotted time makes this an appropriate strategy
Conclusion	• Verify and interpret results with respect to the original problem situation • Generalize solutions and strategies to new problem situations

7th Grade Number and Operations:

Concepts	**Skills and Processes**
Fractional Reasoning	• Use fractions to make comparisons • Use benchmark fractions, decimals and percents to refine estimation skills • Develop strategies for comparing quantities based on given information about how the quantities are related by ratio, fraction, rate, unit rate or percent
Number Operations	• Develop algorithms for computation with positive and negative numbers • Connect integers and operations with them to everyday situations • Develop understanding of multiplication and division of positive and negative numbers • Use unit rates • Compute the slope of a line • Calculate the surface area and volume of prisms
Number Theories	• Develop conceptual models for arithmetic operations with positive and negative integers • Explore the concept of absolute value • Develop meaning for percents greater than 100 and less than 1 • Represent and compare quantities using integers • Explore the use of inverse operations of addition and subtraction, multiplication an division to simplify computations and solve problems

Concepts	**Skills and Processes**
Quantitative Analysis	• Develop a sense of relative size of positive and negative numbers • Order numbers with the > and < relationships and by location on a number line • Develop and use strategies to estimate the results of rational number computations and judge the reasonableness of the results • Develop, analyze and explain methods for solving problems involving proportions, such as scaling and finding equivalent areas • Uses ratios and rates to express "part-to-part" and "whole-to-whole" relationships, and reasons proportionally to solve problem involving equivalent fractions, equal ratios, or constant rates, recognizing the multiplicative nature of these problems in the constant factor of change • Compare and make inferences about populations and quantities • Develop and use strategies for estimating populations

7th Grade Geometry and Measurement:

Concepts	**Skills and Processes**
Measurement	• Measure angles using an angle ruler or a protractor • Measure side lengths using a ruler • Determine and understands length, area and volume (as well as the differences among these measurements), including perimeter and surface area • Use units, square units and cubic units of measure correctly • Calculate volume and surface area of prisms, spheres, cones and cylinders • Calculate volume and surface area of irregular prisms • Draw geometric objects with specified properties, such as side lengths or angle measures • Solve simple problems involving rates and derived measurements for such attributes as velocity and density
Two-Dimensional Similarity	• Explore geometric similarity based on physical appearance • Find correspondence between sides and angles of similar figures • Determine angle relationships in similar figures • Determine side length relationships in similar figures • Develop a definition of mathematical similarity • Reason proportionally in situations with similar figures • Understand relationships among the angles, side lengths, perimeters areas and volumes of similar objects • Create and critique inductive and deductive arguments concerning congruence and similarity
Two-Dimensional Symmetry	• Explore characteristics and properties of line symmetry • Use manipulatives to investigate rotational symmetry

Concepts	**Skills and Processes**
Coordinate Geometry	• Graph figures onto the Cartesian plane using lists of ordered pairs • Generate tables of ordered pairs using transformational rules • Graph transformed images from tables of ordered pairs • Predict how an image will look on a graph by developing an understanding of patterns and transformational rules • Use coordinate geometry to examine special geometric shapes, such as regular polygons or those with parallel or perpendicular sides
Two and Three-Dimensional Shapes	• Precisely describe, classify and understand relationships among types of two and three-dimensional objects using their defining properties • Estimate surface area and volume of irregular three-dimensional figures • Understand the relationship between the volume and dimensions of a prism • Understand how the volumes of cylinders, cones and spheres compare and are related
Transformational Geometry	• Describe sizes, position, and orientations of shapes under informal transformations such as flips, turns, slides and scaling • Examine the congruence, similarity and line or rotational symmetry of objects using transformations • Use algebraic rules to describe geometric transformations • Infer from a transformational rule, whether or not an image will be geometrically similar to the original figure
Subdividing Figures	• Develop understanding about how to subdivide shapes into smaller similar shapes • Infer whether or not a given shape can be subdivided into similar shapes
Scaling	• Determine scale factor given dimensions of two similar figures • Use scale factor to find missing dimensions in similar figures • Investigate relationship between scale factor and area • Use scale maps, diagrams and figures • Reason proportionally with measurements to interpret maps and to make smaller and larger scale drawings • Apply concepts of similar triangles to estimate height of buildings or trees • Estimate distance and area • Scale prisms up and down to meet specific conditions for surface area and/or volume • Given changes in surface area or volume, provide possible scale factors by which the prism was shrunk or enlarged

Concepts	Skills and Processes
Spatial Visualization	• Use geometric models to represent and explain numerical and algebraic relationships • Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science and everyday life • Determine if a flat pattern can fold into a prism • Design flat patterns to fold into prisms of specified dimensions or volume • Determine all different rectangular prisms of a specified volume • Construct three-dimensional figures that meet specific conditions for surface area and volume • Use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume

7th Grade Pattern and Algebraic Reasoning:

Concepts	Skills and Processes
Number Patterns, Relations and Functions	• Represent relationships with tables, graphs in the coordinate plane, and verbal descriptions or equations • Analyze tables, graphs and rules to determine functional relationships • Discover, recognize and describe linear relationships in tables, graphs and equations
Algebraic Symbols	• Develop an initial conceptual understanding of different uses of variables • Explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope • Use symbolic algebra to represent situations and to solve problems that involve linear relationships • Find solutions for unknown quantities in simple linear equations
Mathematical Models	• Understand the connections between tables, graphs, words and equations as different representations of the same relationship • Make decisions about which representations are most appropriate to convey essential information in a given situation
Analyzing Change	• Use graphs, tables, and rules to analyze the nature of changes in quantities in linear relationships and to predict what is not known
Technology	• Use a graphing calculator to enter and display linear equations and tables • Understand the use of the range or window settings to display graphs on a graphing calculator • Use the trace feature of a graphing calculator to find the value of one variable when given the value of the other in a linear situation

7th Grade Communication of Mathematical Ideas:

Concepts	Skills and Processes
Mathematical Language	• Use mathematical language and representations with appropriate accuracy, including numerical tables and equations, simple algebraic equations and formulas, charts, graphs, and diagrams • Use mathematical language to make complex situations easier to understand
Organization	• Organize work, explain facets of a solution orally and in writing • Label drawings • Use other techniques to make meaning clear to the audience
Reasoning	• Exhibit developing reasoning abilities by justifying statements and defending work
Understanding	• Show understanding of concepts by explaining ideas not only to teachers and assessors but to fellow students or younger children
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Implementation	• Use and invent a variety of approaches and understand and evaluate those of others • Invoke problem solving strategies, such as illustrating with sense-making sketches to clarify situations or organizing information in a table • Determine how to break a problem into simpler parts • Solve for unknown or undecided quantities using algebra, graphing, sound reasoning and other strategies • Integrate concepts and techniques from different areas of mathematics • Work effectively in teams when the nature of the task or the allotted time makes this an appropriate strategy
Metacognition	• Monitor and reflect and the process of mathematical problem solving • Build new mathematical knowledge through problem solving
Conclusion	• Verify and interpret results with respect to the original problem situation • Generalize solutions and strategies to new problem situations

8th Grade Number and Operations:

Concepts	Skills and Processes
Number Operations	• Use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions and decimals and algebraic symbols • Understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems • Use the inverse operation to determine unknown quantities in equations • Use and interpret grouping symbols and conventions to compute using the correct order of operations • Perform arithmetic operations involving exponents • Find common factors of polynomial expressions
Number Theories	• Develop an understanding of large numbers and recognize and appropriately use exponential, scientific and calculator notation

8th Grade Geometry and Measurement:

Concepts	Skills and Processes
Coordinate Geometry	• Use coordinate geometry to analyze and explain the Pythagorean theorem, the distance formula and the midpoint formula
Spatial Visualization	• Use geometric models to represent and explain numerical and algebraic relationships • Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science and everyday life

8th Grade Pattern and Algebraic Reasoning:

Concepts	Skills and Processes
Number Patterns, Relations and Functions	• Represent relationships with tables, graphs in the coordinate plane, and verbal descriptions or equations • Analyze tables, graphs and rules to determine functional relationships • Discover, recognize and describe linear, quadratic, exponential and inverse patterns and contrast their properties from tables, graphs and equations • Generate equations by looking at graphs or tables of linear, quadratic or exponential functions

Concepts	**Skills and Processes**
Algebraic Symbols	• Analyze a situation and determine the variables involved and the relationship between the variables • Translate identified variables and relationships into mathematical statements • Explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope in linear functions, maximum and minimum and number of roots in quadratic functions, and intercept in exponential functions • Use symbolic algebra to represent situations and to solve problems that involve linear, quadratic, exponential and inverse relationships • Find solutions for unknown quantities in linear equations • Develop expertise and understanding using factoring to find roots of quadratic equations • Use and understand the quadratic formula for quadratic equations that cannot be factored
Mathematical Models	• Gather and organize data to construct tables, graphs or algebraic rules to use as mathematical models • Use mathematical models to calculate or estimate how change in one variable causes change in the other • Make decisions about which type of function best models a given situation • Develop explanations or predictions about outcomes of the actual situation
Analyzing Change	• Use graphs, tables, and rules to recognize analyze the nature of changes in quantities in linear, quadratic, exponential and inverse variation • Use understanding about exponential growth or decay to develop exponential functions that exhibit specified growth rates or meet other constraints • Explain changes in populations based on understanding of exponential growth and decay patterns
Technology	• Use a graphing calculator to enter and display linear, quadratic and exponential equations and tables • Understand the use of the range or window settings to display graphs on a graphing calculator • Use the trace feature of a graphing calculator to find the value of one variable when given the value of the other in a linear, quadratic or exponential situation • Understand how to use grouping symbols so that a calculator will compute using the correct order of operations • Use graphing calculators to develop the ability to visualize how the graph of a linear, quadratic or exponential equation will appear

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